Translating chord length distribution into bubble size distribution using numerical and analytical backward transforms

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1. Introduction

The bubble sizes and their distribution affect directly to other hydrodynamic behavior of two-phase systems such as flow regime, gas-liquid interfacial area, and mass heat transfer between the phases. However, it is difficult to obtained bubble sizes and their distribution directly. By using the probe technique, the vertical length that bubbles insert the probe, or chord length, can be obtained instead of bubble sizes. In order to get the bubble sizes as well as their distribution, some techniques have to be used. This paper discuss the approaches, which are numerical and analytical backward transform, to estimate bubble size distribution from a set of chord length data obtained by a double-sensor conductivity probe which installed in the air-water vertical loop facility installed in KAERI. The image technique is also used to measure bubble sizes, which used to compare with ones obtained by the probe.

1.1 Correlation between chord length and bubble size

The chord-length density function obtained by assuming that the minimum distance between the center of bubble of radius R and the probe is uniformly distributed can be give by [1,2]

$$P\left(y \mid R\right)=\frac{2r}{R^2}\left|\frac{dr}{dy}\right| \quad (1)$$

Assume that the probability density function (PDF) of bubbles of all sizes touching the probe is P(R). The probability of measuring a particular chord length y for any bubble size R in the system is

$$P\left(y\right)=\int_0^\infty P\left(R\right)P\left(y \mid R\right)dR \quad (2)$$

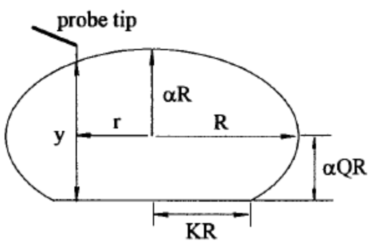

Fig. 1 Truncated ellipsoidal shaped bubble model

There are several axis symmetric shapes used to present bubbles. A typical model employed in this paper is truncated ellipsoidal shapes as shown in figure 1.

The distribution function of chord length is [1]

$$P\left(y \mid R\right)=\begin{cases} \dfrac{y}{2\alpha^2R^2} & 0\le y<2\alpha RQ \\ \dfrac{y}{\alpha^2R^2}\left(y-\alpha RQ\right) & 2\alpha RQ\le y\le \alpha R\left(1+Q\right) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

By using these equations (2) and (3), if P(R) is known then P(y) can be determined and vice versa.

1.2 Numerical backward transform

The algorithm proposed by Clark (1988) is applied in this paper [1]. The procedure expresses the probabilities of finding a chord length y as a product of a triangular matrix with the bubble size probabilities

$$\begin{aligned} W\left(y_i<y<y_{i+1}\right) &=\int_{y_i}^{y_{i+1}} P\left(y\right)dy=\int_{y_i}^{y_{i+1}}\int_0^{R_{\max}} P\left(y \mid R\right)P\left(R\right)dRdy \\ &=\sum_{j=0}^{m-1}\int_{y_i}^{y_{i+1}} P\left(y \mid R_j\right)dyP\left(R_j\right)\Delta R \\ &=\sum_{j=0}^{m-1}C_{ij}P\left(R_j\right)\Delta R \quad (4) \end{aligned}$$

Where

$$y_i=y_{\max}-\left(i+0.5\right)\Delta y,\quad \Delta y=y_{\max}/m \quad \left(0\le i\le m-1\right)$$

$$R_j=R_{\max}-j\Delta R,\quad \Delta R=\frac{R_{\max}}{m}=\frac{y_{\max}}{\alpha\left(1+Q\right)m} \quad \left(0\le j\le m-1\right)$$

1.3 Analytical backward transform

Assume that the probability distribution function of chord length is continuous. Taking the differential of equation (2) and using the definition of P(y|R) in equation (3), the bubble size probabilities can be calculated as follows

$$P_p\left(R\right)=\alpha\left[P_c\left(2\alpha R\right)-2\alpha RP_c^{'}\left(2\alpha R\right)\right] \quad (5)$$

where $P_c(y)$ is the probability function of chord length obtained by curve fitting technique with the data, or by using the Pazen window estimator [2,4].

$$P_c(y) = \frac{1}{nh\sqrt{2\pi}} \sum_{j=1}^{n} e^{-(y-Y_j)^2/2h^2} \quad (6)$$

2. The experimental test facility

The air-water test loop facility was installed at the Korea Atomic Energy Research Institute (KAERI). The experiment uses a double-sensor conductivity probe and high-speed camera to measure local flow parameters. The test facility is shown in figure 2 [3].

The experiment is performed at 1atm and 25^0C. Measurements were obtained at void fraction below 0.45, J_g = 0.4 m/s - 1.9 m/s, and J_f = 0 m/s – 15 m/s.

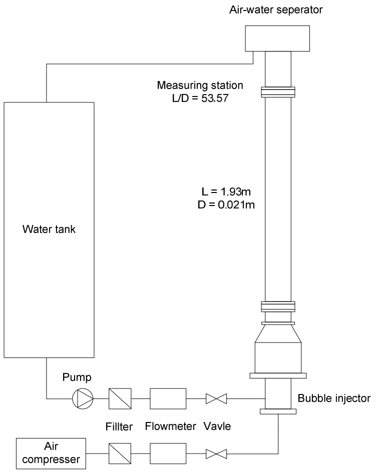

Fig. 2 Double-sensor conductivity probe image (top) and vertical air-water test facility (bottom)

3. Results and discuss

Both of two methods are applied for truncated ellipsoidal shaped bubble with α = 0.5 and k = 0.8. For the analytical backward transform, the ellipsoidal bubbles with same α are also considered.

The chord length distribution as shown in figure 3 is usually irregular and difficult to apply to the numerical backward transform method, which is very sensitive with a subdivision. The results will be irregular and unstable if the number of subdivisions beyond a certain points. Therefore, the chord length data have to fit with a probability function firstly then subdivide into a certain number of intervals. Two kind of fitting functions used are Gaussian and Lognormal. The fitting function is also used in the analytical backward transform. The PDF of bubble size obtained by the method is shown in figure 4.

For the case of J_f = 0m/s and J_g = 0.547 m/s, the chord length data is fitted with the following function

$$P(y) = -0.006 + 2.14e^{-2.94(y-3.408)^2} \quad (7)$$

Then, the equation (5) is applied with α = 0.5, the probability distribution function of bubble size obtained by using analytical backward transform is

$$P(R) = -0.003 + (1.07 + 6.292R)e^{-2.94(R-3.408)^2} \quad (8)$$

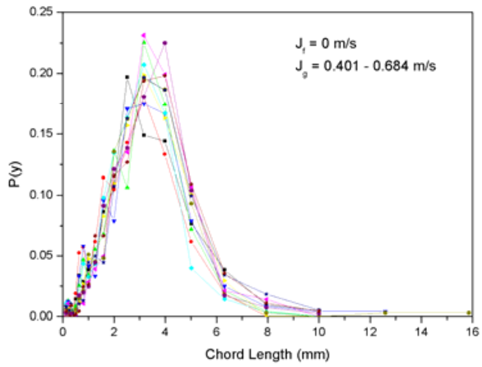

Fig. 3 Probability distribution of chord length

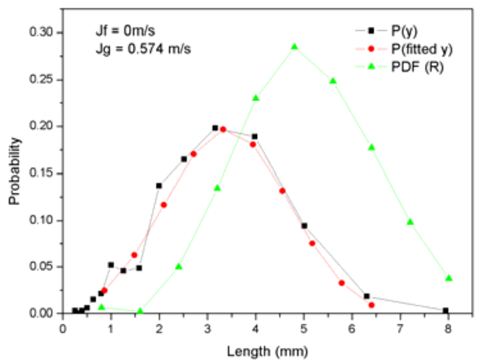

Fig. 4 Probability distributions of chord length and bubble size

4. Conclusions

The probability distribution of bubble size is estimated from the chord length data by the numerical and analytical backward transform. Two methods are employed with the bubble flow, which is modeled with truncated ellipsoidal bubbles. Irregular distributions can occur when using these methods. Therefore, the chord length data were fitted with a function, and a stable distribution can be obtained. Further, the image technique will be applied to verify these results and then improve the algorithms.

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